MATH 4500 – MATRIX GROUPS
SPRING 2024 – SYLLABUS

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Office hours Fri 3:30-4:30pm, Su 3:00-4:00pm, MLT543
Course information will be on Canvas

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Credits/grade.

- 4 credits, Letter grade.
- 3 credits for the lecture/homework/exam component, and 1 additional credit for independent study component.

Prerequisite. multivariable calculus and linear algebra (e.g., MATH 2210-MATH 2220, MATH 2230-MATH 2240, or MATH 1920 and MATH 2940), and familiarity with methods of mathematical proof (as taught, for example, in MATH 3040, MATH 3110, or MATH 3340). MATH4310 or a higher level course in linear algebra is particularly helpful, but not required.

Distribution category. MQR-AS, SMR-AS.

Lectures. 11:15am–12:05pm MWF, MLT203

Description. An introduction to a topic that is central to mathematics and important in physics and engineering. The objects of study are certain classes of matrices, such as orthogonal, unitary, or symplectic matrices. These classes have both algebraic structure (groups) and geometric/topological structure (manifolds). Thus the course will be a mixture of algebra and geometry/topology, with a little analysis as well. The topics will include Lie algebras (which are an extension of the notion of vector multiplication in three-dimensional space), the exponential mapping (a generalization of the exponential function of calculus), and representation theory (which studies the different ways in which groups can be represented by matrices). Concrete examples will be emphasized. Background not included in the prerequisites will be developed as needed.

Textbook.

- John Stillwell. Naive Lie Theory. Springer, 2008. We plan on covering most of the material in this book.
- Kristopher Tapp, Lie groups for undergraduates, second edition, Student Mathematical Libary, volume 79, also available in electronic form from the library.
- Godement, Introduction to the theory of Lie groups, Universitext

These texts are freely available as an e-book through Cornell Library (with a Cornell NetID).

There are many other more advanced textbooks we may draw on. It is important to keep track of what is covered in class, as it will likely diverge from the texts above.

Homework. There will be weekly homework sets (with some exceptions) to be published during the semester. The students will submit their solutions to Gradescope.

INDEPENDENT STUDY. There will be topics to be covered by the students individually, with assistance from the instructor and TA. Students will spend three hours per week, for a total of 40 hours per semester. This will include meetings with the TA or instructor as well as individual study. Students will be asked to write up a summary, and make a presentation.

The topics will be chosen from more advanced material on Lie groups and Lie algebras, possibly related to other subjects such as Mathematical Physics, Number Theory, Analysis and applications.

Exams. There will be two prelims and a final. Prelims will take place in class and the final exam will be scheduled by the University Registrar.

Grading policy. Your course grade will be based on the following with the approximate percentages indicated:

Homework:	16%
Prelim1:	16%
Prelim2:	16%
Final Exam:	36%
Independent Study	16%
Total:	100%

Weeky Schedule. This is subject to a lot of changes. Please keep track of announcements and changes.

Note: items that say "Read" are material you need to read on your own and will not be covered in class (although they may be mentioned briefly).

Week 1: (M 1/22, W 1/24, F 1/26): Read 1.1, 1.6
1.2, 1.3
Week 2: (M 1/29, W 1/31, F 2/2): 1.4
1.5
Read 2.2, 2.6, 2.8, Keith Conrad's notes on Isometries of $\mathbb{R}^n$
https://kconrad.math.uconn.edu/blurbs/grouptheory/isometryR2.pdf
https://kconrad.math.uconn.edu/blurbs/grouptheory/isometryRn.pdf
2.1
Week 3: (M 2/5, W 2/7, F 2/9:
2.2, 2.3
Week 4: (M 2/12, W 2/14, F 2/16):
2.4
2.5
2.6, 2.7
Week 5: (M 2/19, W 2/21, F 2/23)
Read 3.1
3.2, 3.3
Week 6: (M 2/26 (break), W 2/28, F 3/1)
3.4
3.5
3.6
3.7
Week 7: (M 3/4, W 3/6, F 3/8)
3.8
Read 3.9: Chapter 3 Discussions (pp. 71-73)
Read Chapter 4 Preview (p. 74)
4.1
4.2
4.3
4.4
Prelim 1 on Friday 3/8
Week 8: (M 3/11, W 3/13, F 3/15)
4.5
Read 4.6 (pp. 87-90)
Read 4.7: Chapter 4 Discussions (pp. 91-92)
Read Chapter 5 Preview (p. 93)
5.1
5.2
5.3
Week 9: (M 3/18, W 3/20, F 3/22)
M 3/18 : 5.4 , 5.5 (Matrix Lie algebras , their dimensions)
W 3/20 : 5.6 , 5.7 (Complex , quaternion Lie algebras)
F 3/22 : 5.6 , 5.7 (cont'd)
Read 5.8: Chapter 5 Discussions (pp. 113-115)

Read Chapter 6 Preview (p. 116)
Pause in Instruction , Spring Break: 3/16 - 4/6

Week 10: (M 3/25, W 3/27, F 3/29) (Spring break 3/30 to 4/8)
M 3/25 : 6.1 , 6.2 (Normal subgroups, ideals and homomorphisms)
W 3/27 : 6.3 , 6.4, 6.5 (classical Lie algebras, part 1)

Week 11: (M 4/8, W 4/10, F 4/12)
Read 6.7: Chapter 6 Discussion
Read Chapter 7 Preview (p.139)
M 4/8 : 6.6 (classical Lie algebras; end); , 7.1 (logarithm of matrices)
W 4/10 : 7.2, 7.3, 7.4 (exp , log of matrices and tangent spaces)
F 4/12 : 7.5 (non-discrete normal subgroups)
Prelim 2, Friday 4/12

Week 12: (M 4/15, W 4/17, F 4/19)
Read 7.8: Chapter 7 Discussion
Read Chapter 8 Preview (p.160)
M 4/15 : 7.6 (Baker-Campbell-Hausdorff theorem)
W 4/17 : 8.1-8.2 (review of topological spaces)
F 4/19 : 8.3-8.5 (continuity, compactness)

Week 13: (M 4/22, W 4/24, F 4/26)
Read 8.8: Chapter 8 Discussion [includes the "plate trick" in SO(3)]
Read Chapter 9 Preview (p.186)
M 4/22 : 8.6-8.7 (connected, path connected, and simply connected)
W 4/24 : 9.1-9.3 (examples, homomorphisms of Lie aglebras)
F 4/26 : 9.4-9.5 (deformations of paths)

Week 14: (M 4/29, W 5/1, F 5/3 and 5/6) [May 7 is the last day of Spring 2024 classes.]
Read 9.7: Chapter 9 Discussion
M 4/29 : 9.6 (lifting Lie algebra homomorphisms to groups)